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A perturbation solution for the wedge-flow in power-law fluids

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NOMENCLATURE

- f, dimensionless stream function;
- m, exponent in mainstream flow velocity, $U_\infty \sim x^m$;
- n, parameter in constitutive equation for power-law fluids;
- x, distance along the surface;
- α, parameter dependent on n in equation (9);
- β, wedge-parameter in equation (1);
- ϵ, perturbation parameter in equation (3);
- η, similarity variable.

1. INTRODUCTION

Considerable attention has lately been devoted to the study of forced convection boundary layer flows in non-Newtonian power-law fluids. Studies of the corresponding problem of Blasius-flow have been made by Acrivos *et al.* (1960), Defrawi & Finlayson (1972), Lemieux *et al.* (1971) and Roy (1972). Dorfman & Vishnevskii (1972) studied flows with arbitrary pressure-gradients Shah (1961) studied the wedge-flow and obtained similarity solutions for different values of the wedge-parameter β. Acrivos *et al.* (1965) studied afresh Shah's problem and presented an approximate expression for skin-friction by an asymptotic method.

In this communication, we propose to solve the corresponding Falkner-Skan equation for wedge-flows by a perturbation technique. It will be seen that the results thus obtained are fairly reasonable compared with those presented by others.

2. ANALYSIS

The appropriate Falkner-Skan equation is

$$\left.\begin{aligned}&\frac{d}{d\eta}(f'')^n+ff''+\beta\{1-(f')^2\}=0,\\ &\beta = m(n+1)/(2mn-m+1),\end{aligned}\right\} \quad \ldots (1)$$

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subject to the boundary conditions

$$f(0) = f'(0) = 0, \quad f'(\infty) = 1. \qquad \dots \quad (2)$$

Let us assume that the fluid is slightly non-Newtonian and that

$$\left.\begin{aligned} n &= 1+\epsilon \quad (\epsilon \text{ being small}), \\ \text{and} \quad f &= f_0+\epsilon f_1+\epsilon^2 f_2. \end{aligned}\right\} \qquad \dots \quad (3)$$

Substituting from (3) into (1) and equating the co-efficients of different powers of ϵ to zero we get the following equations for f_0, f_1 and f_2.

$$\left.\begin{aligned} & f_0'''+f_0 f_0''+\beta_0(1-f_0'^2) = 0, \\ & f_1'''+f_0 f_1''-2\beta_0 f_0' f_1'+f_0'' f_1+\beta_1(1-f_0'^2) \\ & \qquad +f_0'''(1+\log f_0'') = 0, \\ & f_2'''+f_0 f_2''-2\beta_0 f_0' f_2'+f_0'' f_2+f_1'''(1+\log f_0'')+f_1 f_1''-\beta_0 f_1'^2 \\ & \qquad -2\beta_1 f_0' f_1'+\beta_2(1-f_0'^2)+f_0'''\{f_1''/f_0''+\log f_0''+ \\ & \qquad +\tfrac{1}{2}(\log f_0'')^2\} = 0, \\ & f_i(0) = f_i'(0) = 0, \quad f_0'(\infty) = 1, \\ & f_1'(\infty) = f_2'(\infty) = 0, \end{aligned}\right\} \qquad \dots \quad (4)$$

where

$$\beta_0 = \frac{2m}{m+1}, \quad \beta_1 = \tfrac{1}{2}\beta_0(1-2\beta_0), \quad \beta_2 = -\beta_0\beta_1.$$

3. Solutions

Equations (4) have been numerically solved on an electronic computer The results are presented in table 1. So,

$$f''(0) = f_0''(0)+\epsilon f_1''(0)+\epsilon^2 f_2''(0) \qquad \dots \quad (5)$$

Table 1. The values of $f_0''(0)$, $f_1''(0)$ and $f_2''(0)$ for different β_0

β_0	0	$\frac{1}{2}$	1
$f_0''(0)$	0.4696	0.927680	1.232588
$f_1''(0)$	0 112497	−0.169612	−0.685606
$f_2''(0)$	0.019918	0 266235	0 970367

4. Comparison with Existing Results

Case 1 : $\beta_0 = 0$

This is the most extensively studied case. Shah (1961) gave an exact solution, Lemieux *et al* (1971) gave an exact solution together with one based on variational principle. Also they gave formulae similar to (5) for pseudoplastic fluids ($n < 1$), namely,

$$\text{exact solution}: \quad f''(0) = 0{\cdot}4752 + 0{\cdot}1696\,\epsilon, \tag{6}$$

$$\text{variational principle}: \quad f''(0) = 0{\cdot}5057 + 0{\cdot}1648\,\epsilon \tag{7}$$

We found a quadratic fit, by the method of least squares, of the results given by Shah. It is

$$f''(0) = 0{\cdot}4787 + 0{\cdot}0975\,\epsilon - 0{\cdot}0041\,\epsilon^2. \tag{8}$$

In table 2 we compare the results given by the equations (5)–(8) with those of Shah (1961).

Table 2. Values of $f''(0)$ for $\beta_0 = 0$

ϵ	Shah (1961)	Eq. (6)	Eq. (7)	Eq. (8)	Roy
−0 5	0 4341	0.3904	0 4215	0.4289	0.4183
0.0	0 4696	0 4752	0 5057	0 4787	0 4696
0 5	0.5258	0 5600	0.5899	0 5264	0 5308
1 0	0 5766	0.6448	0.6741	0.5720	0 6020
1 5	0 6188	0 7296	0 7583	0 6156	0 6832
2.0	0 6540	0.8144	0.8425	0 6572	0 7743

Case 2 : $\beta_0 = \frac{1}{2}$ *and* 1

Acrivos *et al* (1965) presented the following approximate relationship based on the asymptotic solutions of equations (1) and (2) as $\beta \to 0$ and as $\beta \to \infty$

$$f''(0) \simeq \left\{ \alpha^{n+1} + \frac{2}{3}\, \frac{m(n+1)^2}{n(1-m+2mn)} \right\}^{1,} \tag{9}$$

where $\alpha = \alpha(n)$ are the values of $f''(0)$ for $\beta = 0$ calculated by Shah (1961), namely the entries in the second column of table 2.

We compare our results with those of Shah (1961) and Acrivos *et al* (1965) for $\beta_0 = \frac{1}{2}$ and 1 in table 3.

Table 3. Values of $f''(0)$ for $\beta_0 = \frac{1}{2}$ and 1

	Acrivos *et al* (1965)		Shah (1961)	Roy	
c	$\beta_0 = \frac{1}{2}$	$\beta_0 = 1$	$\beta_0 = 1$	$\beta_0 = \frac{1}{2}$	$\beta_0 = 1$
−0.5	1.1826	2.209	2.187	1.0790	1.8202
0.0	0.9419	1.246	1.233	0.9277	1.2326
0.5	0.8942	1.081	—	0.9094	1.1346
1.0	0.8844	0.981	0.973	1.0243	1.5263

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